GAIN ADAPTATION IN SLIDING MODE CONTROL OF ROBOTIC MANIPULATORS

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ABSTRACT

In this paper, an adaptation scheme is proposed to adapt the gains of the classical SMC to overcome some of the problems faced in practical implementations of motion control systems. A Lyapunov function is selected for classical SMC design and MIT rule is used for gain adaptation. The criterion that is minimized for gain adaptation is selected as sum of squares of control signal and sliding function. This novel approach is also applied to control of a scara type robot manipulator.

KEYWORDS: Sliding Mode Control, Gain Adaptation, MIT Rule, Scara Robot

INTRODUCTION

In most of the motion control applications, the system dynamics or the parameters may change with time. For this kind of systems, the robust control techniques should be chosen to overcome these changes.

A powerful control technique for alleviating the problem of parameter changes is the use of Variable Structure System (VSS) theory with a Sliding Mode Control (SMC) [1]. It is also a technique easy to use since only the bounds on the parameters need to be known.

As a result of the classical SMC design by selecting a Lyapunov function, the control input is calculated as the sum of the equivalent control and an additional term [2]. Equivalent control is the control that makes the derivative of the sliding function equal to zero. The additional term which is directly proportional to the sign of the sliding function is used to compensate the deviations from the sliding surface.

The classical SMC suffers mainly from two disadvantages. The first one is the chattering which is high frequency oscillations of the controller output. The second one is that the complete knowledge of the plant dynamics is needed in the calculation of the equivalent control. In literature, there are some suggestions to solve these problems. The well known chattering elimination technique is the use of a saturation function [3]. On the other hand, computational burden of the equivalent control can be avoided with using least square (LS) estimation or recursive LS [4].

In this paper, a gain adaptation scheme is proposed which directly results in chattering-free control action. A different estimation mechanism to compute the equivalent control is also used.

The proposed approach is applied to control of a scara type robot which is a two degrees of freedom planar robot manipulator. The dynamics consists of nonlinear coupled equations [5]. The kinematics and inverse kinematics are used for trajectory generation.

The paper is organized as follows: Next section is devoted to the SMC approach and its implementation via the estimation of equivalent control. In section III, a gain adaptation technique for SMC is proposed, which uses the well-known MIT rule [4]. Section IV, consists of simulation examples of a robotic manipulator and aims to asses the performance of the proposed adaptation technique. Section V concludes the paper.

SLIDING MODE CONTROL (SMC)

The Variable Structure System (VSS) theory has been applied to control nonlinear processes [6,7]. One of the main features of this approach is that one only needs to drive the error to a “switching surface” or “sliding surface”, after which the system is in “sliding mode” and will not be affected by any modeling uncertainties and/or disturbances.

Most of the systems can be written in the state space form representation as,

$$\dot{x}(t) = f(x) + Bu(t) \tag{1}$$

where x is (nx1) state vector, f is a nonlinear function of x, B is (nxm) input gain matrix, and u is (mx1) input. For this kind of systems, generally, sliding surface-S (mx1) is selected [2] as,

$$S(x,t) = G(x^r(t) - x(t)) = \phi(t) - S_a(x) \tag{2}$$

where

$$\phi(t) = G\,x^r(t), \qquad S_a(x) = G\,x(t) \tag{3}$$

and x^r represents the reference state vector and G (mxn) is the slope matrix of the sliding surface. In this kind of representations, the system states have to include the derivatives for second and higher order systems.

The aim in SMC is to force the system states to the sliding surface. Once the states are on the sliding surface, the system errors converge to zero with an error dynamics dictated by the design of the matrix G.

Classical Sliding Mode Controller (CSMC)

The method described in this section is based on the selection of a Lyapunov function. The control should be chosen such that the candidate Lyapunov function satisfies Lyapunov stability criteria [2,8]. A Lyapunov function for each independently controlled axis of a motion control system is selected as,

$$V_i = \frac{S_i^T S_i}{2} \qquad \text{i=1..m} \tag{4}$$

where m is the number of the axes. Clearly, this function is positive definite.

It is aimed that the derivative of the Lyapunov function is negative definite. This can be attained if one can assure that

$$\frac{dV_i}{dt} = -D_i\, S_i^T\, sign(S_i) \tag{5}$$

D is a scalar positive gain value. Taking the derivative of the (4), and equating this to (5), one will obtain the following equation,

$$S_i\,\frac{dS_i}{dt} = -D_i S_i\, sign(S_i) \tag{6}$$

By taking the time derivative of (2) and using the plant equation,

$$\frac{dS_i}{dt} = \frac{d\phi_i}{dt} - \frac{\partial S_{ai}}{\partial x_i}\frac{dx_i}{dt} = \frac{d\phi_i}{dt} - G_i(f_i(x) + B_i u_i) \tag{7}$$

is obtained. By putting the (7) into the (6), the control input signal can be obtained as,

$$u_i(t) = u_{eqi}(t) + D_i\,(G_i B_i)^{-1} sign(S_i) \tag{8}$$

where, $u_{eq}(t)$ is the equivalent control and can be written as,

$$u_{eqi}(t) = -(G_i B_i)^{-1}\left(G_i f_i(x) - \frac{d\phi_i(t)}{dt}\right) \tag{9}$$

Estimation of the Equivalent Control

If the knowledge of *f(x)* and *B* matrices is very poor, then the equivalent control calculated will be too far off from the actual equivalent control. In the literature a number of approaches are proposed for the estimation of u_{eq}, rather than calculating it.

In this paper, a recent approach [2] is used keeping in mind the fact that the equivalent control is actually the average of the total control [1]. An averaging filter for calculation of the equivalent control can be designed as,

$$\tau \dot{\tilde{u}}_{eq_i}(t) + \tilde{u}_{eq_i}(t) = u_i(t) \tag{10}$$

or

$$\tilde{u}_{eq_i}(t) = \frac{1}{\tau\, p + 1} u_i(t) \tag{11}$$

where $\tilde{u}_{eq\,i}$ is an estimate for $u_{eq\,i}$ and $p = d/dt$. The average of the control is computed and fed back to calculate the control to be applied in the next control cycle. This method requires less knowledge about the system, and thus alleviates some of the problems resulting from the uncertainties in the plant.

Equation (11) is actually a low-pass filter. The value of $1/\tau$ gives the cut-off frequency. The logic behind the designing a low pass filter is that low frequencies determine the characteristics of the signal, and high frequencies result from unmodeled dynamics.

Now, one can choose the control as follows,

$$u_i(t) = \tilde{u}_{eq_i}(t) + D_i\,(G_i B_i)^{-1} sign(S_i) \tag{12}$$

where $\tilde{u}_{eq_i}(t)$ is defined as in (11).

ADAPTIVE SLIDING MODE CONTROL (ASMC)

Classical SMC causes to chattering. The sign of the additional term ($D_i\left(G_{\acute{Y}}B_{\acute{Y}}\right)^{-1}$) to equivalent control changes its sign when S_i rings around zero. If one takes this additional term as constant, chattering is faced. When it reaches the sliding surface, this term should be minimized. An adaptation scheme to minimize the control effort and sliding function is proposed using the MIT rule. The criterion which is minimized is chosen as,

$$J_i = \frac{1}{2}\left(S_i^T S_i + u_i^T u_i\right) \tag{13}$$

To make J_i small it is reasonable to change the parameters (D_i) in the direction of the negative gradient of J_i, i.e.,

$$\frac{dD_i}{dt} = -\gamma_i \frac{\partial J_i}{\partial D_i} \tag{14}$$

Substituting (13) into (14) and taking the partial derivative,

$$\frac{dD_i}{dt} = -\gamma_i\, S_i \frac{\partial S_i}{\partial D_i} - \gamma_i\, u_i \frac{\partial u_i}{\partial D_i} \tag{15}$$

Taking the partial derivative of (8) with respect to D_i,

$$\frac{\partial u_i}{\partial D_i} = \left(G_i B_i\right)^{-1} sign(S_i) \tag{16}$$

The partial derivative of S_i with respect to D_i can be calculated as,

$$\frac{\partial S_i}{\partial D_i} = \frac{\partial}{\partial D_i}\left(G_i\left(x_i^r - x_i\right)\right) = -\frac{\partial}{\partial D_i}\left(G_i x_i\right) \tag{17}$$

Substituting (8) into (1) and taking the integral of it, (17) can be written as,

$$= -\frac{\partial}{\partial D_i}\int_{t_0}^{t}\left(G_i f_i + G_i B_i u_{eqi}(\zeta) + D_i sign(S_i(\zeta))\right)d\zeta \tag{18}$$

$$\frac{\partial S_i}{\partial D_i} = -\int_{t_0}^{t} sign(S_i(\zeta))d\zeta \qquad (19)$$

Substituting (19) and (16) into the (15),

$$\frac{dD_i}{dt} = \gamma_i\, S_i \int_{t_0}^{t} sign(S_i(\zeta))d\zeta - \gamma_i^{'} u_i sign(S_i) \qquad \text{i=1,..,m.} \qquad (20)$$

is obtained. When selecting the Lyapunov function, there was a restriction on D_i being a positive value. Therefore, calculated D_i passed through a limiter as,

$$D_i = \begin{cases} D_i & D_i > 0.05 \\ 0.05 & D_i \le 0.05 \end{cases} \qquad (21)$$

The lower value of D_i is selected as 0.05 instead of 0, in order to suppress the error arising from the estimation of the equivalent control.

ROBOTIC APPLICATION

Robot Dynamics

The work presented in this paper considers a robot with a two degrees of freedom planar manipulator. A detailed description of the dynamics of the robot is given in [5]. By selecting the state vectors as,

$$x_{11} = \theta_1 \qquad x_{12} = \dot{\theta}_1 \qquad x_{21} = \theta_2 \qquad x_{22} = \dot{\theta}_2 \qquad (22)$$

where θ_i's (i=1,2) are the joint angles (see Fig.1). The state space representation can be given as,

$$\begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a_{12} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & h_1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} u_1$$

$$\begin{bmatrix} \dot{x}_{21} \\ \dot{x}_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & h_2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} u_2 \qquad (23)$$

where,

$$a_{12} = C\left(\frac{4}{3}\sin(x_{21})x_{22} + \frac{2}{3}\sin(x_{21})x_{12}\right) + C\cos(x_{21})\sin(x_{21})x_{12}$$

$$a_{22} = C\left(\frac{-4}{3}\sin(x_{21})x_{12} - \frac{2}{3}\sin(x_{21})x_{22}\right) + C\left(-\cos(x_{21})\sin(x_{21})x_{22} - 2\cos(x_{21})\sin(x_{21})x_{12}\right)$$

$$h_1 = C\left(\frac{2}{3}\sin(x_{21})x_{22}\right), \qquad h_2 = C\left(\frac{-10}{3}\sin(x_{21})x_{12}\right) \qquad \text{and} \qquad C = \frac{9}{16 - 9\cos^2(x_{21})}$$

Now, the plant equations can be written in a more convenient matrix form as,

$$\dot{x}_1 = f_1(x_1, x_2) + B_1\, u_1 \qquad \dot{x}_2 = f_2(x_1, x_2) + B_2\, u_2 \qquad (24)$$

where,

$$x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \qquad x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ C \end{bmatrix} = B_2 \qquad (25)$$

and the definition of f_1 and f_2 are obvious from (23).

Simulation Results

In simulation studies carried out, with proper selection of gain matrices, the system perfectly follows the desired trajectory for both of the controllers designed above. In all simulations, it is intended to control a scara type robot manipulator to follow such a square trajectory in X-Y plane as presented in Fig 2.

The control is implemented with both constant and adapted gains. Instead of calculating the equivalent control the estimation technique given by (11) is used in both cases. The G and D matrices are selected as, $G_1 = [1 \quad 1] = G_2 \quad D_1 = 1 = D_2$.

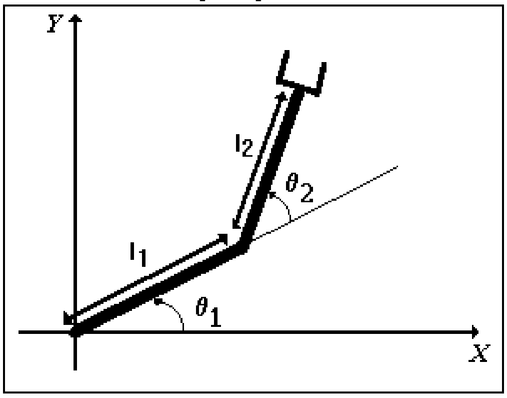

Figure 1. Robot Kinematics

The simulation results for classical SMC (CSMC) are presented from in Figs 4-6. In this case, the manipulator follows the desired trajectory, but the controller outputs make chattering as shown in Fig6.

The error for adaptive SMC is much better then the classical one. The controller outputs make some chattering at the beginning and then becomes smooth when it reaches the sliding surface. The results are presented in Figs 7-9.

The essential tuning parameters in adaptation are γ_i's in equation (20). When selecting a value for γ_i, there are two criterion; adaptation capability and stability. While a low value causes to low adaptation capability, a high value may cause to instability. As a result, a sufficiently large value that does not make the system unstable should be chosen. The results presented here are obtained by $\gamma_{1,2} = 5.0$ and $\gamma_{1,2}^{'} = 1.0$. Only some of the simulation results are presented in this paper because of space unavailability. But the other results are similiar to the presented ones.

CONCLUSIONS

In this paper, an adaptation scheme using MIT rule is applied to adapt the gains of the classical SMC which is designed by selecting a Lyapunov function. The aim is to eliminate the chattering and to reduce the error. Therefore, the cost function of MIT rule is selected as sum of squares of the control signal and the sliding function.

The real reason of the chattering in classical SMC is the unnecessary control effort. As a result of the design, the controller is obtained as equivalent control plus an additional term. The additional term is necessary when the system is not on the sliding surface. This term takes the system on to the sliding surface. The equivalent control is enough to keep the system on the sliding surface when the system on the sliding surface. Therefore, the additional term should be minimized when the system reach on the sliding surface. This minimization can be achieved by minimizing the multiplicative gain D_i .This is the crucial point of using the adaptation. The additional term should be minimized, but actually not to be made zero because the system may keeps away from the sliding surface any reasons such as external disturbance or sharp changes in the reference signal. It is meaningful to use a limiter with a minimum of 0.05 for D_i .

The simulation results presented in this paper indicate that the suggested approach has considerable advantages compared to the classical one and is capable of achieving a good chatter-free trajectory following performance without an exact knowledge of plant parameters. These characteristics make it a promising approach for motion control applications.

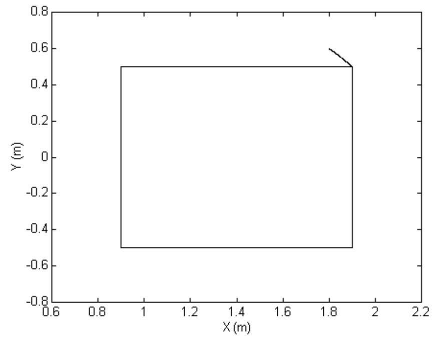

Figure 2. The Motion on X-Y Plane

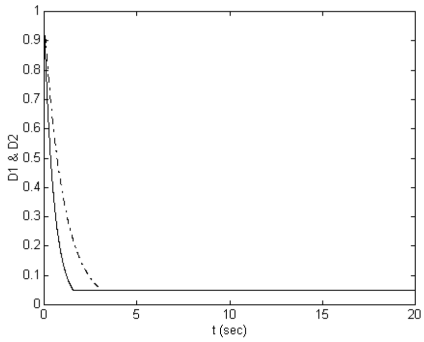

Figure 3. Parameters D1 & D2 for ASMC

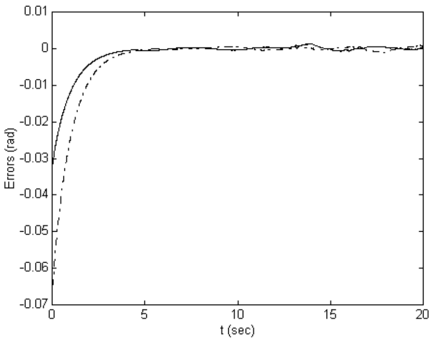

Figure 4. Angle errors for CSMC

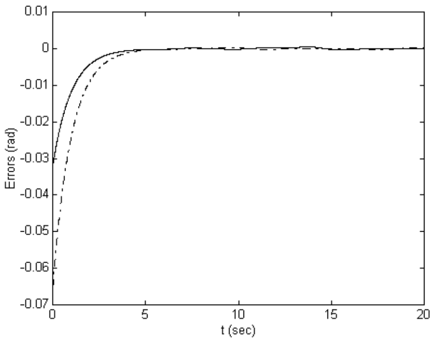

Figure 7. Angle errors for ASMC

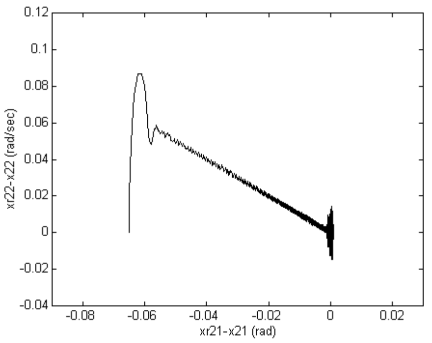

Figure 5. Phase Plane 2 for CSMC

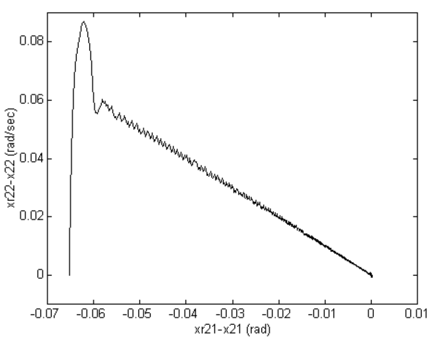

Figure 8. Phase Plane 2 for ASMC

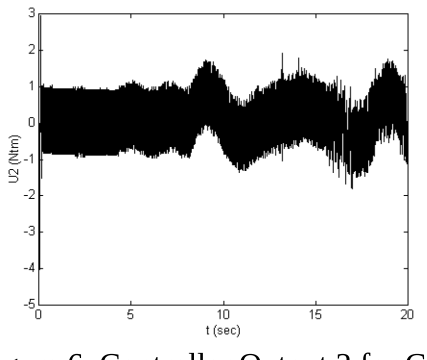

Figure 6. Controller Output 2 for CSMC

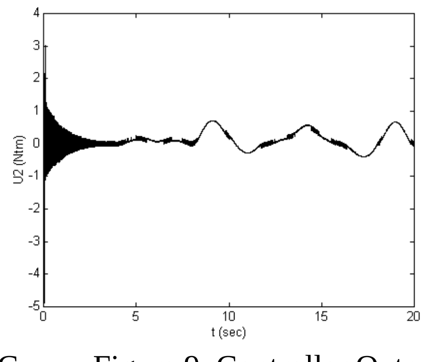

Figure 9. Controller Output 2 for ASMC

REFERENCES

1. Utkin, V.I., *Sliding Modes in Control Optimization*, Springer-Verlag, 1981.

2. Ertuğrul, M., Okyay K., Þabanoviç A., “Various VSS Techniques on the Control of Automated Guided vehicles”, *IEEE International Symposium on Industrial Engineering, ISIE’95*, Greece, 1995

3. Hung J. Y., “Variable Structure Control: A survey”, *IEEE Transaction on industrial Electronics*, Vol.40, no.1, February, 1993.

4. Aström K. J. and Wittenmark B., *Adaptive Control* Addison-Wesley Publishing 1989.

5. Spong M.W. and Vidyasagar M., *Robot Dynamics and control*, John Wiley & Sons Inc. 1989.

6. Young, K.D., “Controller design for a manipulator using theory of variable structure systems”, *IEEE Transactions on Systems Man, and Cybernetics,* Vol. SMC-8 pp.210-218, Feb. 1978.

7. Slotine, J.J., Sastry, S.S., “Tracking Control of nonlinear systems using sliding surfaces with applications to robot manipulators”, *International Journal of Control*, Vol 38. pp.465-492, 1983.