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Navigation of non-communicating autonomous mobile robots with guaranteed connectivity

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SUMMARY
We consider the connectivity of autonomous mobile robots. The robots navigate using simple local steering rules without requiring explicit communication among themselves. We show that using only position information of neighbors, the group connectivity can be sustained even in the case of bounded position measurement errors and the occlusion of robots by other robots in the group. In implementing the proposed scheme, sub-optimal solutions are invoked to avoid an excessive computational burden. We also discuss the possibility of deadlock which may bring the group to a standstill and show that the proposed methodology avoids such a scenario in real-life settings.

KEYWORDS: Mobile robots; Multi-robot systems; Navigation; Robot localization; Swarm robotics.

1. Introduction

The navigation of a mobile robot as a single agent which follows a given trajectory is a well-studied problem today. Several tools from the classical control theory can be applied and satisfactorily good results can be obtained both theoretically and practically. However, there are many tasks in which the use of just one mobile robot is inadequate. In such cases, multiple mobile robots are expected to be more successful when they behave as a group. Control, coordination, and navigation of mobile robot groups is an active area of research in which several important results have been achieved in the last two decades.

In this paper, we study an important aspect of navigation of multiple autonomous mobile robots, namely, their connectivity. Loosely speaking, connectivity of a robot group, or simply a connected group, implies that motion of one robot may cause all other robots in the group to change their positions accordingly. Of course, the information about the motion of one robot must be available to other robots either directly or indirectly so that they can change their positions by appropriate motions. There are several ways to achieve this. For example, this information can be delivered by the moving robot to other robots through a communication channel. Another way, when such a communication channel is not available, is to gather this information in an indirect manner, through sensors. The sensors may be various, such as visual, laser, or ultrasonic. In any case, connectivity is based on the propagation of positional changes of robots throughout the whole group.

The navigation of autonomous robots may be a secondary task of a group or its primary mission, depending on the application. The voyage of a group of mine digging robots from one site to another is an example of the former, in which navigation appears as a secondary task in the travel of robots. However, navigation is the primary task of robots in missions such as defense patrols or underwater exploration. In both cases, navigation as a connected group is of vital importance, since it corresponds to the unity of the group. Thus, connectivity and its maintainability are fundamental concepts in almost any study regarding the decentralized group motion.

The idea of group behavior of autonomous robots also has a background in, and is inspired by, nature. Indeed, one can observe such group movements in some species, especially in schooling fish, flocking birds, and the colonies of bees and ants. A large number of studies are available in the literature which incorporate the group motion of autonomous agents. One of the first efforts to model species exhibiting group behavior was given for flocking birds by Reynolds,[1] with the assertion that such group behaviors arise as a result of simple principles related to the position and velocity of each member of the group. An important application of this idea in discrete time was given by Vicsek *et al.*.[2] Since then, the concept of cooperative motion has evolved greatly. The formation of robot groups[3,4] and the utilization of potential functions and artificial forces to accomplish a group behavior[5–7] have been widely studied. Some methodologies rely on limited communication between robots for a desired group task.[8,9] An extensive discussion of the initial studies conducted in this area and development of basic concepts can be found in ref. [10].

Many authors have employed graph theory[11–18] and potential fields[19] in the studies related to connected navigation and the group behavior of mobile robots. Tanner *et al.*[11,12] assumed that a state vector consisting of position and velocity is measurable and every robot can sense any other robot in the group without any restrictions. These studies and also the study by Jadbabaie *et al.*[20] *assume* that group connectivity or communication during the period of motion is a prerequisite for the success of their methods. There are only a few studies which aim to maintain connectivity without relying on information exchange or

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communication between robots.[21–23] But their algorithmic methodologies assume that robots are points and are designed to work only in $\mathbb{R}^2$ with perfect measurements via sensors.

Graph theoretic approaches to maintain connectivity of mobile agents are mainly based on the maximization of the second smallest eigenvalue (Fiedler value) of the Laplacian matrix of graph.[15–17, 24] Even if this maximization can be done in a distributed manner as suggested by De Gennaro and Jadbabaie,[15] this does not eliminate the necessity of communication between robots. For example, the method introduced in ref. [15] requires some data to be obtained from neighbors to update components of the supergradient of the Laplacian matrix that are computed locally. Another example of mobile group navigation using limited communication can be found in ref. [16], where decentralized feedback controllers for multiple nonholonomic robots are proposed, which provide collision avoidance and satisfy agent-specific goal configurations at the cost of exponential complexity. A comprehensive discussion and theoretical framework of controlling graph connectivity in mobile networks is given by Zavlanos *et al.*[17] with various optimization approaches and applications to rendezvous, flocking, and formation control. In all these works[15–17] and ref. [19], the control input to each mobile agent arises as a solution to (or optimization of) an algebraic system in which desired group behaviors, such as connectivity, trajectory following, and collision avoidance, are already embedded; whereas the methodology presented here allows independent control inputs but applies some constraints thereafter to guarantee group connectivity.

In this paper we develop a methodology for the navigation of autonomous robot groups which preserve group connectivity. We assume that the robots have relative position sensors but no communication capabilities. This means that each robot can only acquire distances to its neighbors and their relative angles. To make our work more realistic, we also assume that their position sensors are of limited range and have bounded measurement errors. We consider our robots as capable of moving in any direction, and the navigation space free of obstacles. The lack of communication among robots leads to a strategy that is conservative in preserving links in the group. Therefore, the resulting method is more useful for applications where the mission involves transporting the group along a trajectory, rather than coverage of wide areas.

The methodology proposed in a previous work by the authors[25] results in the navigation of a robot group having dynamic topology using only limited-range relative position sensors with guaranteed connectivity. This work did not address issues such as measurement errors, occlusion of robots, and deadlock. Recently, the analysis in ref. [26], where an ad hoc method was proposed to resolve a possible deadlock, was taking measurement errors into account. Nevertheless, a proof on the avoidance of a deadlock was missing. Here we reformulate the navigation strategy to eliminate the possibility of a deadlock and extend our results so as to include the possibility that robots may be occluded by others.

In the following section, we describe our robot model and define the connected navigation problem. Section 3 includes the proposed methodology, gives the basic theorem on connectivity, and explains how deadlock is avoided. Section 4 presents a sub-optimal solution to the optimization problem for the movement of robots in their local coordinates. The proposed scheme and the sub-optimal solution are tested by various simulations in Section 5. Lastly, conclusions about the study are offered in Section 6.

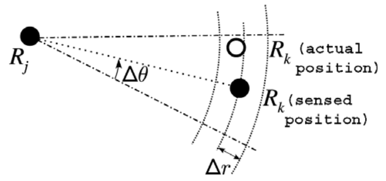

Fig. 1. Angular and radial measurement errors in $\mathbb{R}^2$. $\Delta\theta$ and Δr are the bounds on the measurement error.

2. Problem Formulation

The robots in this study are assumed to be identical. Each robot has the capability of moving in all directions (i.e. the robots are *omni-directional*), and are equipped with limited-range relative position sensors. The sensors can measure both distances and relative angles of robots within their measurement range. The working space can be either two-dimensional or three-dimensional. These sensors have a known degree of accuracy. We assume that the sensing capability is continuous and available equally in all directions, but the sensor results can bear both angular and radial measurement errors. These errors are bounded by positive scalars $\Delta\theta$ and Δr respectively. This is depicted in Fig. 1 for a two-dimensional case.

It is important to note that sensing other robots means obtaining information about the position of robots in the neighborhood via relative position sensors. We refer to such a mutual visibility between robots as a *link*. However, such a link does not require any explicit communication or information exchange between robots. As robots move, as long as they maintain visibility with their neighbors, they can avoid separation from other robots, even if they do not communicate with them. Note that sensing other robots does not imply recognizing a specific robot. In other words, the robots have no IDs or labels.

We denote a group of autonomous mobile robots with links based on their sensing neighborhood as $\mathcal{G}$ and the individual robots as R_i $(i = 1, \ldots, N)$. Note that the subscripts are arbitrary and for the sake of analysis only. Considering the robots $R_1, \ldots, R_N$ as the vertices and the links between them as the edges of an undirected graph, this graph is *connected* if there is a path from any robot to another robot in the group through links.[24] Hence, without loss of any rigor, we can say that the group $\mathcal{G}$ is connected whenever the graph corresponding to $\mathcal{G}$ is connected. Conversely, a group which has at least one pair of robots having no in-between path is *disconnected*.

Since we assume that the position sensing ranges of robots are limited and the total number of robots in the group can be large, a robot may not sense all other robots in the group. We call the set of robots sensed by R_i as the subgroup $\mathcal{S}_i$. So there are N such subgroups of $\mathcal{G}$ and if $\mathcal{G}$ is connected, $\mathcal{S}_i$ $(i = 1, \ldots, N)$ are nonempty sets.

We denote the radius of the spherical region having R_i in its center and containing the robots in $\mathcal{S}_i$ as $d_{\max}$. In other

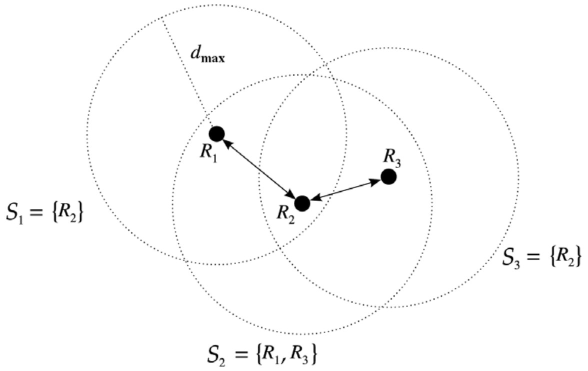

Fig. 2. A group of three robots (R_1, R_2, R_3) and corresponding subgroups (S_1, S_2, S_3). The distance between R_1 and R_3 is larger than d_{max}, hence $R_1 \notin S_3$ and $R_3 \notin S_1$.

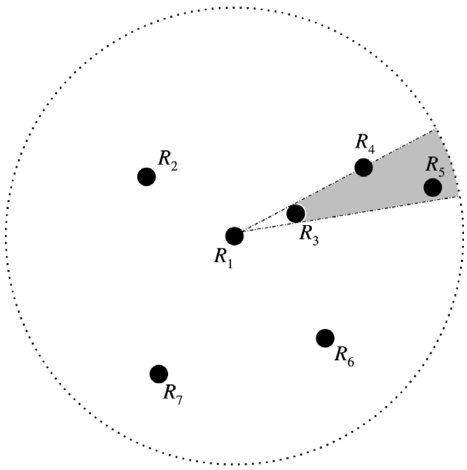

Fig. 3. R_3 occludes R_4 and R_5 from R_1.

words, d_{max} is the maximum sensing distance for each robot. If d_{max} is large enough so that the robots can sense even the farthest member of the group, then $\mathcal{G}$ will be connected. However, one faces nontrivial and more interesting cases for robot groups of a large number of individuals having short sensing ranges and are spread over a relatively large area.

Figure 2 depicts a group consisting of three robots. In this configuration $R_2 \in \mathcal{S}_1$ and $R_1 \in \mathcal{S}_2$, so R_1 and R_2 are linked. The links between R_2 and R_3 are formed likewise. Note that the robot R_2 has the position information of both R_1 and R_3, but R_1 and R_3 cannot sense each other as the distance between them is larger than d_{max}. We assume that sensing is always mutual, that is, if a robot R_i senses any other robot R_j, then R_j has the position information of R_i too.

In implementing position measurement, which might be performed using any kind of ultrasonic, laser, or vision-based sensors, it is inevitable that some robots might occlude others. In such a case, occluded robots are not sensed by a robot, say R_1 (hence, they are not in $\mathcal{S}_1$), although their distances to R_1 are less than d_{max}. Figure 3 depicts an example of occlusion, where position measurements of R_4 and R_5 cannot be accomplished becuase R_3 prevents R_4 and R_5 from being "in sight" of R_1. Consequently, whenever occlusion occurs, the positions of the occluded robots cannot be taken into account in the computation of the local movement at that time instant. Note that the mutuality of position sensing is also valid under occlusions.

Having these sensing limitations and assuming that a set of robots initially form a connected group, our objective in this work is to develop a decentralized steering methodology that allows navigation of the group while preserving its connectivity without requiring any explicit information exchange between the robots.

Once connectivity is assured, the target or navigation trajectory of the mission need not be known by all group members. In fact, it suffices if only one robot has this information.[22] We call this robot the *leader* of the group and denote it as R_N. Nevertheless, the leader has the same physical properties and capabilities as the other robots. The only difference is that the trajectory to be followed by the group is given to R_N. In fact, the leadership of the group is *hidden*. None of the robots recognize the leader as a distinguished group member. In other words, if R_N is sensed by robot R_j, i.e. $R_N \in \mathcal{S}_j$, R_j can only see it as one of its neighbors and the leadership of R_N does not affect the local steering strategy of R_j. In the following part of the paper, we consider the group of N robots consisting of one leader, R_N, and $N-1$ *followers*, R_j ($j = 1, \ldots, N-1$).

3. Autonomous Motion

Our goal is to develop a methodology for simple autonomous robots such that a large group of them can move as a connected group. We assume that the robots update the position information about their neighbors at every Δt seconds. Also, to take measurement errors on the distance into account, we define a positive scalar d_m as

$$d_m \stackrel{\text{def}}{=} d_{max} - \Delta r,$$

where Δr is the bound on the distance measurement error with $d_{max} > \Delta r > 0$. We denote the position of a robot R_i at time t as $X_i(t)$, $i = 1, \ldots, N$. Since all robots in the group steer autonomously, we will set up local moving rules for each robot. Below we propose a local steering strategy that is inspired by the preliminary study given by Reynolds.[1] At each sampling instant, the robots acquire the positions of other robots in their sensor range. While the leader R_N moves along a predefined trajectory, each follower robot R_j, $j = 1, \ldots, N-1$, determines a local target location for itself. This motion is most conveniently described in terms of a coordinate system attached to R_j. Obviously, R_j is at the origin of this local coordinate system. Let $x(t)$ denotes the position vector in local coordinates. We will use a notation such that the superscripts in x relate the coordinate frame to a robot, and the subscripts in x indicate which robot's position it is. For example, x_k^j represents the position vector of R_k in the coordinate frame of R_j. For the robots in $\mathcal{S}_i$, $i = 1, \ldots, N$, we have

$$\|x_k^i(t)\| = \|X_k(t) - X_i(t)\| \leq d_m, \qquad k = 1, \ldots, M$$

where M is the number of robots in $\mathcal{S}_i$. Next, we propose the steering strategy to be employed by each robot using the positions of other robots in its subgroup.

3.1. Local steering strategy

According to the notation given in the previous section, $x_i^i(t+\Delta t)$ is the location which R_i is aiming at (for the time instant $t+\Delta t$) in R_i's own coordinate system at time t. For any $x_i^i(t+\Delta t)$, we define two complementary subsets of $\mathcal{S}_i$ as

$$\mathcal{S}_{ip} = \left\{R_p \in \mathcal{S}_i \mid [x_i^i(t+\Delta t)]^{\mathrm{T}} x_p^i(t) \leq 0\right\},$$
$$\mathcal{S}_{iq} = \left\{R_q \in \mathcal{S}_i \mid [x_i^i(t+\Delta t)]^{\mathrm{T}} x_q^i(t) > 0\right\}.$$

If the displacement of R_i to $x_i^i(t+\Delta t)$ takes R_i closer to a robot, then this robot appears in $\mathcal{S}_{iq}$, otherwise it will be a member of $\mathcal{S}_{ip}$. Using $\mathcal{S}_{ip}$ and $\mathcal{S}_{iq}$, we can state the following theorem on group connectivity.

Theorem 1. *Consider a group $\mathcal{G}$ of N autonomous mobile robots which are connected at $t = 0$. If the motion of the robots are subject to the constraints*

$$\|x_i^i(t+\Delta t)\| \leq \frac{1}{2}\left(d_m - \max_{x_p^i(t)\in\mathcal{S}_{ip}} \|x_p^i(t)\|\right) \quad (1)$$

and

$$\|x_i^i(t+\Delta t)\|^2 \leq \min_{x_q^i(t)\in\mathcal{S}_{iq}} \left\{[x_i^i(t+\Delta t)]^{\mathrm{T}} x_q^i(t)\right\} \quad (2)$$

for $i = 1, \ldots, N$, the group preserves its connectivity for $t > 0$.

Proof. Note that the position of each robot in $\mathcal{S}_i$ can constrain the motion of R_i either via Ineq. (1) or Ineq. (2) according to whether this robot appears in $\mathcal{S}_{ip}$ or $\mathcal{S}_{iq}$. Let R_a and R_b be any two robots within their mutual sensing range, that is $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$ at time t.

First, suppose that $R_b \in \mathcal{S}_{ap}$ and $R_a \in \mathcal{S}_{bp}$. Then it follows from Ineq. (1)

$$2\|x_a^a(t+\Delta t)\| + \max_p \|x_p^a(t)\| \leq d_m \quad (3)$$

and

$$2\|x_b^b(t+\Delta t)\| + \max_p \|x_p^b(t)\| \leq d_m. \quad (4)$$

Noting that $\max_p \|x_p^a(t)\| \geq \|x_b^a(t)\|$, $\max_p \|x_p^b(t)\| \geq \|x_a^b(t)\|$, and $\|x_b^a(t)\| = \|x_a^b(t)\|$, we obtain from Ineqs. (3) and (4),

$$\|x_a^a(t+\Delta t)\| + \|x_b^b(t+\Delta t)\| + \|x_b^a(t)\| \leq d_m.$$

Further, by triangle inequality, we get

$$\|x_a^a(t+\Delta t) - [x_b^a(t) + x_b^b(t+\Delta t)]\| \leq d_m. \quad (5)$$

Note that the term $x_b^a(t) + x_b^b(t+\Delta t)$ is the position of R_b at time $t+\Delta t$ as expressed in the local coordinate frame attached to R_a at time t. Therefore, Ineq. (5) shows that the distance between the robots R_a and R_b will not be larger than d_m at time $t+\Delta t$.

Next, we assume that $R_b \in \mathcal{S}_{aq}$ and $R_a \in \mathcal{S}_{bq}$. In this case, we have to proceed using the constraint in Ineq. (2), namely,

$$\|x_a^a(t+\Delta t)\|^2 \leq [x_a^a(t+\Delta t)]^{\mathrm{T}} x_b^a(t). \quad (6)$$

Since

$$\left\|x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2}\right\|^2 = \|x_a^a(t+\Delta t)\|^2 + \left\|\frac{x_b^a(t)}{2}\right\|^2 - [x_a^a(t+\Delta t)]^{\mathrm{T}} x_b^a(t), \quad (7)$$

using Ineq. (6), we obtain

$$\left\|x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2}\right\| \leq \frac{\|x_b^a(t)\|}{2}. \quad (8)$$

Note that $x_b^a(t) = -x_a^b(t)$ and using the triangle inequality along with Ineq. (8), it follows that

$$\begin{aligned}
&\|x_a^a(t+\Delta t) - [x_b^a(t) + x_b^b(t+\Delta t)]\| \\
&= \left\|\left(x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2}\right) - \left(x_b^b(t+\Delta t) - \frac{x_a^b(t)}{2}\right)\right\| \\
&\leq \left\|x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2}\right\| + \left\|x_b^b(t+\Delta t) - \frac{x_a^b(t)}{2}\right\| \\
&\leq \|x_b^a(t)\| \\
&\leq d_m,
\end{aligned} \quad (9)$$

which asserts the link between R_a and R_b at time $t+\Delta t$ in the same way as done by Ineq. (5).

To complete the proof, we have to also analyze the cases where $R_b \in \mathcal{S}_{aq}$ while $R_a \in \mathcal{S}_{bp}$, and $R_b \in \mathcal{S}_{ap}$ while $R_a \in \mathcal{S}_{bq}$. Without loss of generality, we consider only the former, since the proof for the latter can be obtained by an interchange of subscripts a and b only.

In other words, the motion of R_a and R_b will be constrained by Ineqs. (6) and (4) respectively. Similar to Eq. (7), we can write

$$\left\|x_a^a(t+\Delta t) - x_b^a(t)\right\|^2 = \|x_a^a(t+\Delta t)\|^2 + \left\|x_b^a(t)\right\|^2 - 2[x_a^a(t+\Delta t)]^{\mathrm{T}} x_b^a(t).$$

In view of Ineq. (6), we get

$$\|x_a^a(t+\Delta t) - x_b^a(t)\| \leq \|x_b^a(t)\|. \quad (10)$$

Therefore, Ineq. (4) with Ineq. (10) yields

$$2\|x_b^b(t+\Delta t)\| + \|x_a^a(t+\Delta t) - x_b^a(t)\| \leq d_m$$

or

$$\|x_a^a(t+\Delta t) - x_b^a(t) - x_b^b(t+\Delta t)\| \leq d_m - \|x_b^b(t+\Delta t)\|. \quad (11)$$

Hence, the validity of Ineq. (5) is maintained in this case too.

The results in Ineqs. (5), (9), and (11) show that any two robots $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$ sensing each other at time t will still be linked when they move to their new locations at $t + \Delta t$. Hence, we conclude that if the group is connected at $t = 0$, it will also be connected for $t > 0$. □

Note that if a robot is occluded by another robot in $\mathcal{S}_i$, the number of robots in $\mathcal{S}_i$ might decrease. Nevertheless, this situation does not disturb the overall connectivity, as the existence of the occluding robot itself is the evidence of the connection between R_i and the occluded robot. Also, the fact that constraints (1) and (2) restrict the maximum steering distances of robots for each sampling period Δt brings the advantage of avoiding inappropriate large velocities.

As long as the constraints (1) and (2) are satisfied, following a given navigation trajectory, formation control and other mission-oriented tasks can be accomplished by using potential function approaches or minimizing cost functions.[3,4] Therefore, in view of Theorem 1, the following local steering strategy assures the connectivity of the robot group which is composed of follower robots and a leader in navigation.

Local Steering Strategy. Subject to constraints (1) and (2),

- *The follower robots R_j ($j = 1, \ldots, N-1$) move toward a target location $x_j^j(t + \Delta t)$, which minimizes the cost function, $J(x_j^j(t + \Delta t))$, related to the positions of robots in $\mathcal{S}_j$.*
- *The leader R_N follows the navigation trajectory.*

Several types of cost functions can be used in implementing the local steering strategy. An example may be given as

$$J(x_j^j(t + \Delta t)) = \max_k \|x_j^j(t + \Delta t) - x_k^j(t)\|, \qquad (12)$$

which makes the jth robot try to decrease the distance to the farthest robot that it senses. Another possible approach could be to force the robots to keep their distances with the robots in their subgroups as close to a desired distance as possible. Denoting the desired distance as d_0 ($d_0 < d_m$), we can define a suitable cost function for each follower robot R_j as

$$J(x_j^j(t + \Delta t)) = \sum_{k=1}^{M} \left(\|x_j^j(t + \Delta t) - x_k^j(t)\| - d_0 \right)^2. \qquad (13)$$

Note that both Eqs. (12) and (13) are defined in terms of local coordinates to ensure a distributed algorithm. Although they happen to be convex functions, this is not a requirement from the point of view of connectivity. The choice of cost function depends on mission requirements. One can consider fixed as well as time-varying cost functions. These can incorporate the position information of all or only some of the neighbors. Further, there is no reason why each member of the group does not minimize a different cost function.

3.2. Deadlock-free motion

In the previous section, we employed two constraints, namely (1) and (2), on the motion of robots to assure their connectivity. At this point, one may raise the question whether these constraints can lead to a situation where none of the robots can move. Such a situation is called a *deadlock* and its avoidance is of crucial importance for the applicability of the proposed method in real-life implementations.

In view of constraints (1) and (2), a deadlock occurs whenever $\|x_i^i(t + \Delta t)\| = 0$, $i = 1, \ldots, N$. In such a case, since all follower robots lock each other, the leader cannot progress on its predefined trajectory without breaking connectivity. This will eventually cause the whole group to remain permanently immobile. Such deadlocks due to the immobility of robots, for example, have been discussed in ref. [27].

We state in the following theorem that a deadlock cannot occur, provided that the group navigates under the local steering strategy.

Theorem 2. *Let $\mathcal{G}$ be an initially connected group navigating freely in $\mathbb{R}^n$ and consisting of finite number of robots that move according to the local steering strategy. Assume that $J(x_i^i(t + \Delta t)) = \tilde{J}(\|x_i^i(t + \Delta t) - x_j^i(t)\|)$ is an increasing function of $\|x_i^i(t + \Delta t) - x_j^i(t)\|$ at $\|x_i^i(t + \Delta t) - x_j^i(t)\| = d_m$ for all j such that $R_j \in \mathcal{S}_i$. Then, for any robot $R_a \in \mathcal{G}$, we have*

$$\max_i \|x_i^a(t)\| < d_m \qquad \text{as } t \to \infty \qquad (14)$$

where x_i^a's are the position vectors of robots in $\mathcal{S}_a$.

Proof. From Theorem 1, we already know that

$$\max_i \|x_i^a(t)\| \leq d_m$$

for $0 \leq t < \infty$. In order to prove the theorem, one must show that $\|x_i^a(t)\| = d_m$ cannot hold forever.

Consider the smallest convex set $\mathbb{S}(t)$ that includes X_i, $i = 1, \ldots, N$ at time t. Since N is finite and the navigation space is whole $\mathbb{R}^n$, this means that $\mathbb{S}(t) \subset \mathbb{R}^n$, and also all of its vertices are occupied by robots.

Let y be any point in $\mathbb{S}(t)$, with y^o being its coordinates in the reference frame attached to some vertex robot R_o. Since $\mathbb{S}(t)$ covers all robots in the group and R_o is at the vertex of $\mathbb{S}(t)$, it follows that $[y^o]^{\mathrm{T}} x_i^o(t) > 0$ for all $i = 1, \ldots, N$. Therefore, there is a feasible set for the minimization of $J(x_o^o(t + \Delta t))$, where $\mathcal{S}_{op} = \emptyset$, and hence the constraint (1) is not active. Since constraint (2) is the only active constraint and $\tilde{J}$ is assumed to be increasing in $\|x_o^o(t + \Delta t) - x_i^o(t)\|$, it follows that the minimization of J over this feasible set will yield a local target location for R_o so that $\|x_o^o(t + \Delta t)\| \neq 0$ and $\|x_o^o(t + \Delta t) - x_i^o(t)\| < d_m$. In other words, R_o will be driven into the convex set $\mathbb{S}(t)$ if there is a robot $R_x \in \mathcal{S}_o$ with $\|x_x^o\| = d_m$.

Now, suppose there exists a robot R_b such that $\|x_b^a(t)\| = d_m$ at $t = T_0 \geq 0$. If R_b is the only robot in $\mathcal{S}_a$, we can make the same argument as in the previous paragraph and conclude that the local target for R_a given by the minimization of J would drive R_a *toward* R_b, and thus assuring $\|x_b^a(t + \Delta t)\| < d_m$. The persistency of the distance between R_a and R_b being

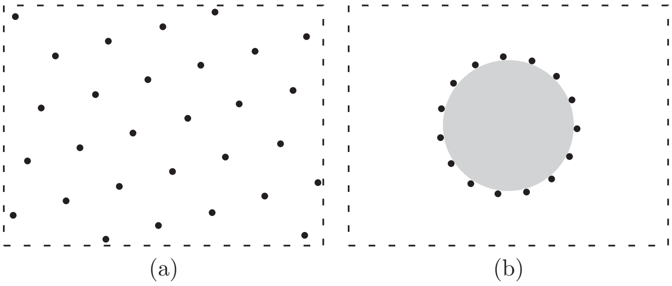

Fig. 4. Two examples of deadlock. (a) Infinitely many robots covering $\mathbb{R}^2$. (b) Fifteen robots in a proper subset of $\mathbb{R}^2$ (gray area does not belong to the navigation space).

d_m, that is

$$\|x_b^a(t)\| = d_m \qquad \text{for } t \geq T_0 \geq 0 \tag{15}$$

requires that there must be at least one other robot in $\mathcal{S}_a$, say R_c, so that the set $\mathcal{S}_{ap}$ is nonempty for any direction of $x_a^a(t+\Delta t)$ and Ineq. (1) yields $\|x_a^a(t+\Delta t)\| = 0$ for $t \geq T_0$. Obviously, R_c must be at a distance of d_m to R_a too.

Using a similar argument, we can deduce the same conditions for all robots R_i $(i = 1, \ldots, N)$, i.e., $\mathcal{S}_{ip}$ is nonempty for any direction of $x_i^i(t+\Delta t)$. But this condition means that none of the robots are at a vertex of $\mathbb{S}(t)$ and therefore it contradicts our assumption that N is finite. Hence, we conclude that Eq. (15) cannot hold for any two robots R_a and R_b. This proves the validity of Ineq. (14). □

Both of the cost functions (12) and (13) (for $d_0 < d_m$) satisfy the requirements of the theorem. Therefore, they will be suitable for deadlock-free navigation.

Considering these cost functions in view of Theorem 2, we can directly state the following corollary.

Corollary 1. *For a group of robots that are connected at $t = 0$ and subject to the local steering strategy with the cost function (12) or (13), the occurrence of a deadlock necessarily requires that either the navigation space is a proper subset of $\mathbb{R}^n$, or there are infinitely many robots in the group.*

As an example of a deadlock, one can think of a connected group of robots that are covering the surface of a sphere whose navigation space is the surface of this sphere. Alternatively, a navigation space as $\mathbb{R}^2$ (or $\mathbb{R}^3$) with a circular (or spherical respectively) hole in it may lead to a deadlock. This sort of deadlock is depicted Fig. 4.

Theorem 2 presents an important basis for the guaranteed navigation of the group. That is, if one of the robots in the group is the leader and is given a trajectory to be followed, the leader will have the freedom to progress through its trajectory without breaking connectivity no matter how the trajectory is shaped. This is clear from the fact that

$$d_m - \lim_{t\to\infty} \max_i \|x_i^N(t)\| > 0$$

and the strict inequality assures a non-zero distance that the leader can move at each sampling time. The rest of the group will then follow the leader accordingly.

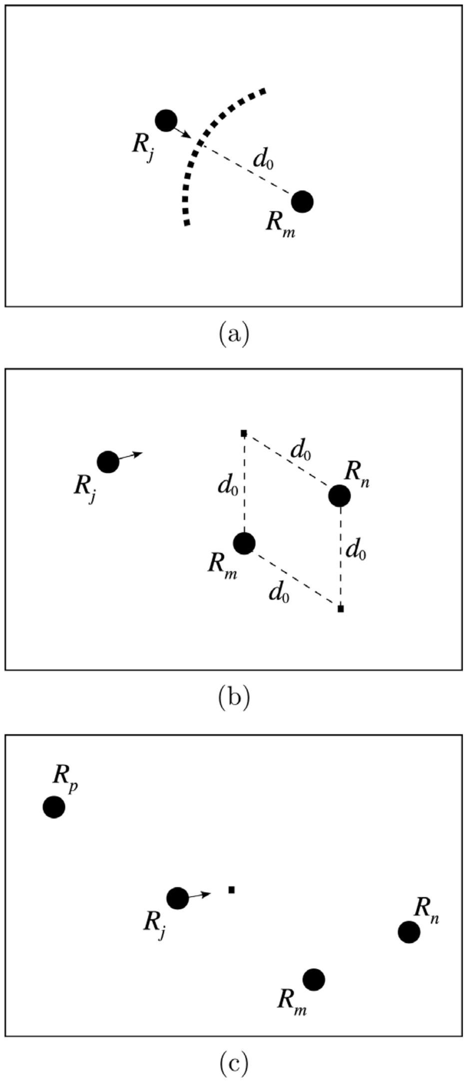

Fig. 5. Examples of local target locations in $\mathbb{R}^2$. (a) Only one robot in $\mathcal{S}_j$, hence infinitely many local target candidates (the nearest is chosen). (b) Two robots in $\mathcal{S}_j$ giving two symmetric local target candidates. (c) Three robots in $\mathcal{S}_j$ with unique local target.

4. Sub-Optimal Solution to Local Steering Problem

The robots in this study are supposed to be quite simple and limited devices especially from the computational point of view. Our purpose is to provide a decentralized control methodology that can be applied to such simple robots to lead to satisfactorily good group navigation. Below we propose a gradient-descent-based iterative method to reduce computational burden in the implementation of the local steering strategy.

The minima of the cost function given in Eq. (13) are the locations where each follower robot R_j aims to reach at each sampling time. The minimization of Eq. (13) subject to the constraints in Ineqs. (1) and (2) requires higher computational power as the number of robots in $\mathcal{S}_j$ increases. First consider some simpler cases where the optimal target locations can be easily characterized by inspection.

When there is only one robot, say R_m, in a subgroup $\mathcal{S}_j$, the solution set for $x_j^j(t+\Delta t)$ is an arc in $\mathbb{R}^2$ or $\mathbb{R}^3$, with a diameter d_0. So it consists of an infinite number of points (Fig. 5(a)). The movement direction is either toward R_m if the distance between R_j and R_m is larger than d_0, or away from R_m if the distance is smaller than d_0. Obviously, no movement is required if R_j is already lying on an optimal point at that time.

When there are only two robots, say R_m and R_n, in $\mathcal{S}_j$, one may consider different cases as far as optimal points are concerned. If the distance between R_m and R_n is larger than or equal to $2\,d_0$, the optimal point is unique in both $\mathbb{R}^2$ and $\mathbb{R}^3$, and it is at the center of the line segment connecting R_m and R_n. Otherwise in $\mathbb{R}^2$ there are two points lying symmetrically on each side of the line connecting R_m and R_n so that $J(x_j^j(t+\Delta t)) = 0$ as shown in Fig. 5(b). Nevertheless, only the point closer to robot R_j will satisfy constraint (2). In $\mathbb{R}^3$, number of optimal points are infinite and they lie in a semicircle whose center is the center of the line segment connecting R_m to R_n. This line segment is also normal to the semicircle of solution points.

Finding the local target is relatively easy when only one or two robots are sensed by each follower robot at a time. However, whenever three or more robots are in a subgroup $\mathcal{S}_j$, the minimization of J given by Eq. (13) is more troublesome. At the extremum points, we have

$$\frac{\partial J}{\partial x_j^j} = \sum_{k=1}^{M} \frac{\partial}{\partial x_j^j} \left(\|x_j^j(t+\Delta t) - x_k^j(t)\| - d_0 \right)^2 = 0 \quad (16)$$

which results in a nonlinear set of equations. The solution of the system in Eq. (16) may not be unique. After solving this system, the solution points must then be tested to see if their values are minimum. If multiple minima are present, we can find the global minimum by evaluating the cost function given in Eq. (13) at these points.

It should be noted that the optimal points are computed for each follower robot R_j at every sampling time. The location of optimal points depends on the positions of robots in the subgroup $\mathcal{S}_j$. Since the sensed robots in $\mathcal{S}_j$ also move autonomously, these local targets will be updated every Δt seconds, possibly before reaching them. Since Ineqs. (1) and (2) constrain the magnitude of $x_j^j(t+\Delta t)$, they will not be violated as the robots are moving toward their local targets. Hence, the solution will only provide a direction to optimal points because the solution will be updated before R_j reaches that location. Also, it should be noted that even if the optimal local target is not reached by the robots, Theorem 2 holds as long as they move in the same direction as the optimal target.

This fact can be exploited to introduce an iterative method to implement the local steering strategy in a sub-optimal way. Rather than solving for the minimum points of the cost function, each robot R_j can move in the direction of the negative gradient of the cost function evaluated at the position of R_j for each sampling instant. That is,

$$x_j^j(t+\Delta t) = x_j^j(t) - \gamma \left. \frac{\partial J(x_j^j(t+\Delta t))}{\partial x_j^j(t+\Delta t)} \right|_{x_j^j(t+\Delta t) = x_j^j(t)}, \quad (17)$$

```
read sensor data;

if leader,
  read given trajectory;
  assign  direction := value from trajectory;
          motion_size := predefined max value (physical limit);
else
  calculate J;
  assign  direction := -grad J;
          motion_size := value from evaluation of grad J;
end if

classify sensed robots as p and q using direction;

apply motion limits;
   motion_size := max value satisfying inequalities in Theorem 1;

if motion_size > predefined max value,
  assign motion_size := predefined max value;
end if

move in the direction as far as motion_size;
```

Fig. 6. Pseudo-code for the sub-optimal algorithm.

where $\gamma > 0$ is a positive gain, and x_j^j is the position vector of R_j in its local coordinates. From Eq. (13) it follows that

$$\frac{\partial J(x_j^j(t+\Delta t))}{\partial x_j^j(t+\Delta t)} = 2 \sum_{k=1}^{M} \left(\|x_j^j(t+\Delta t) - x_k^j(t)\| - d_0 \right) \times \frac{x_j^j(t+\Delta t) - x_k^j(t)}{\|x_j^j(t+\Delta t) - x_k^j(t)\|}, \quad (18)$$

with M being the number of robots in $\mathcal{S}_j$. Further, since $x_j^j(t) = 0$, from Eqs. (17) and (18) we obtain

$$x_j^j(t+\Delta t) = 2\gamma \sum_{k=1}^{M} \left(\|x_k^j(t)\| - d_0 \right) \frac{x_k^j(t)}{\|x_k^j(t)\|}. \quad (19)$$

The gain γ in Eq. (19) should be treated as a parameter by which one can choose the distance of the local target so as to satisfy the constraints in Ineqs. (1) and (2) rather than a constant to be determined *a priori*. In that respect, there is no reason why γ must be kept constant during navigation. The application of Eq. (19) is much simpler than solving the system in Eq. (16). It gives the direction of the next movement, and the movement in this direction is realized only if it satisfies the inequalities in Ineqs. (1) and (2). We should point out that although the method introduced above is similar to a potential function approach, the minimization of the cost function does not serve as a guarantee for connectivity. Rather it is merely a tool to achieve the motion of the group. The connectivity is already assured by Theorem 1 without any reference to cost functions. Pseudo-code of the algorithm is given in Fig. 6. In the next section we test the proposed methodology with the

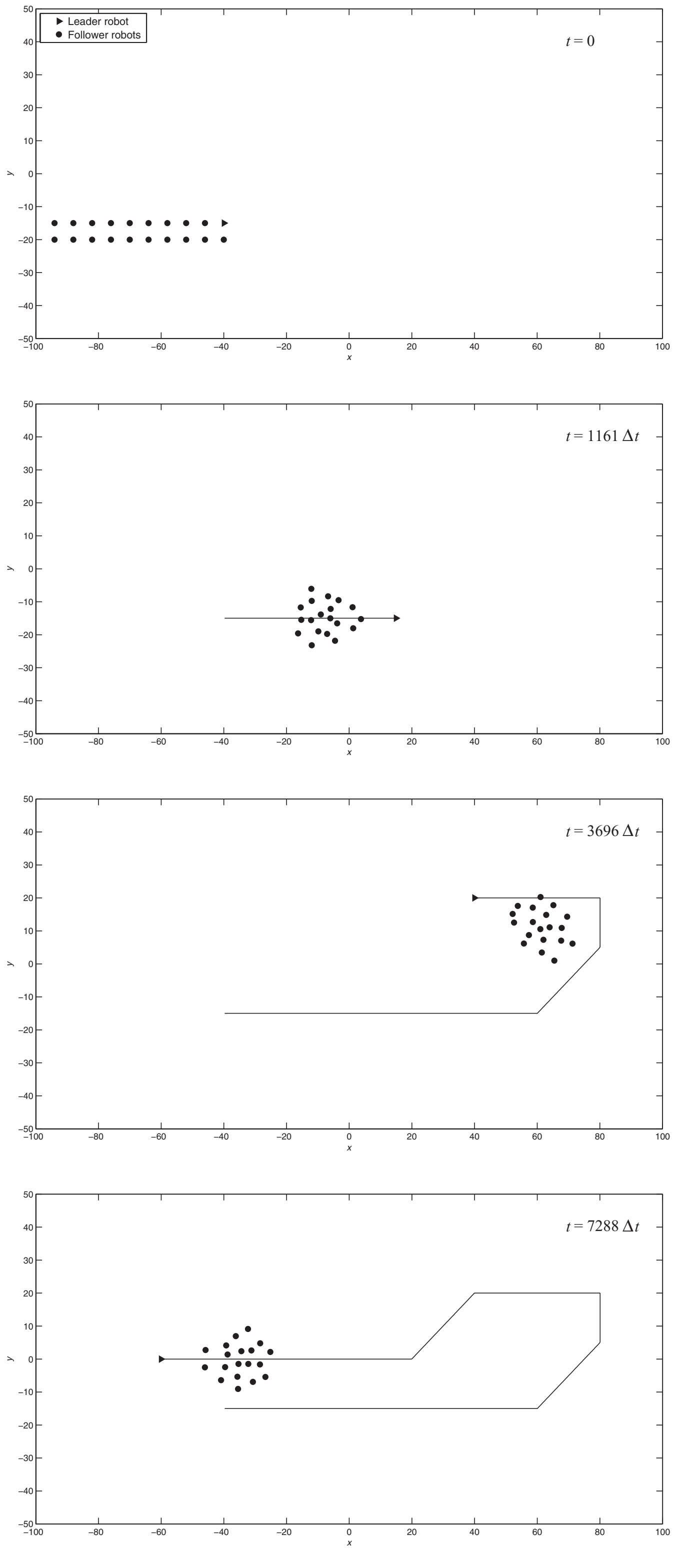

Fig. 7. Four snapshots of navigation of 20 robots ($d_{\max} = 15$, $d_0 = 8$). The solid line denotes the trajectory of the leader.

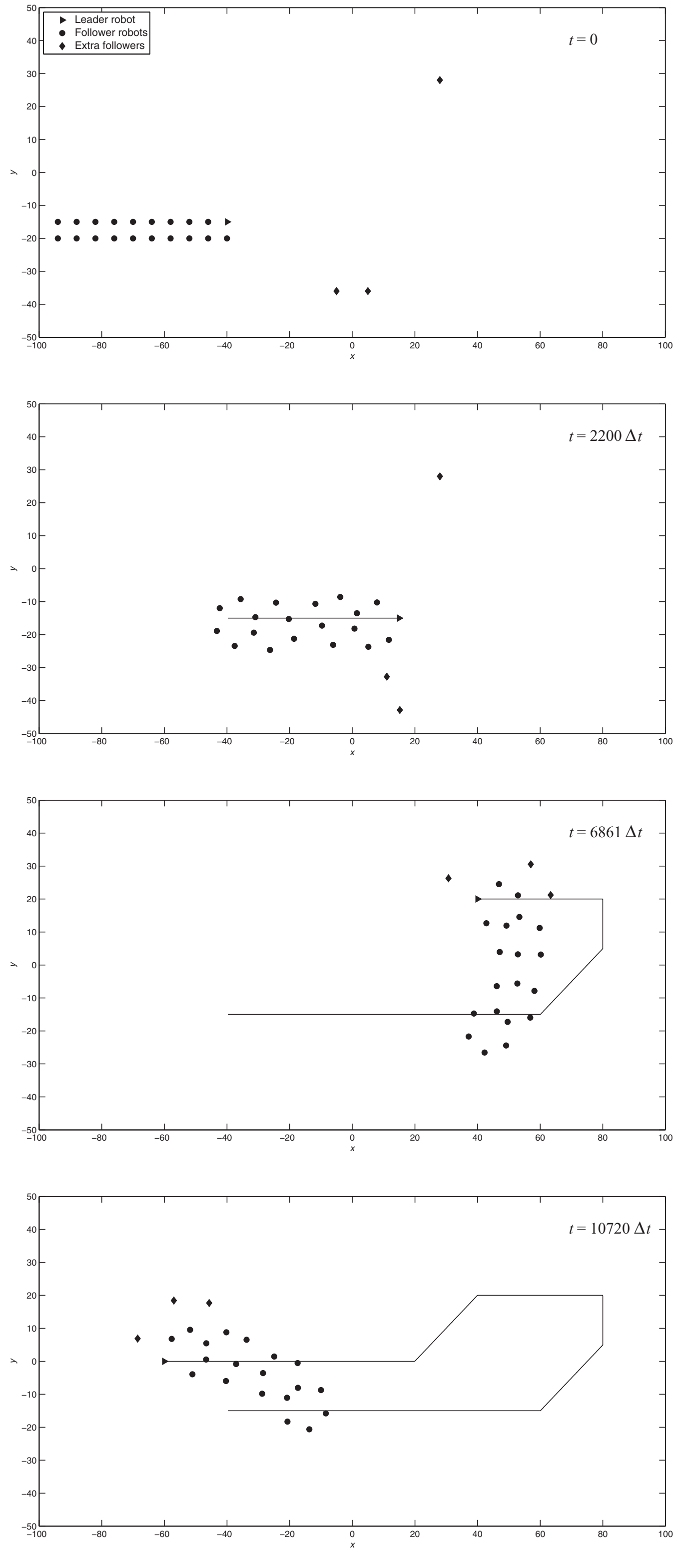

Fig. 8. Four snapshots of the navigation of 20 robots ($d_{\max} = 15$, $d_0 = 11$). Three extra robots, shown as diamond-shaped, join the group on the way.

local steering of robots in the direction of negative gradient as given in Eq. (17).

5. Simulation Results

We illustrate the theoretical results of the previous sections with computer simulations. The robot group, composed of disk-shaped robots having omni-directional motion capability, is assumed to navigate in $\mathbb{R}^2$. The sensor range ($d_{\max}$) was 15 units. The bounds on the measurement errors were $\Delta\theta = 12^\circ$ for angle and $\Delta r = 0.03\ d_{\max}$ for distance measurement. Diameter of the robots was 1.5 units. For each robot in the group, this value and its position information in

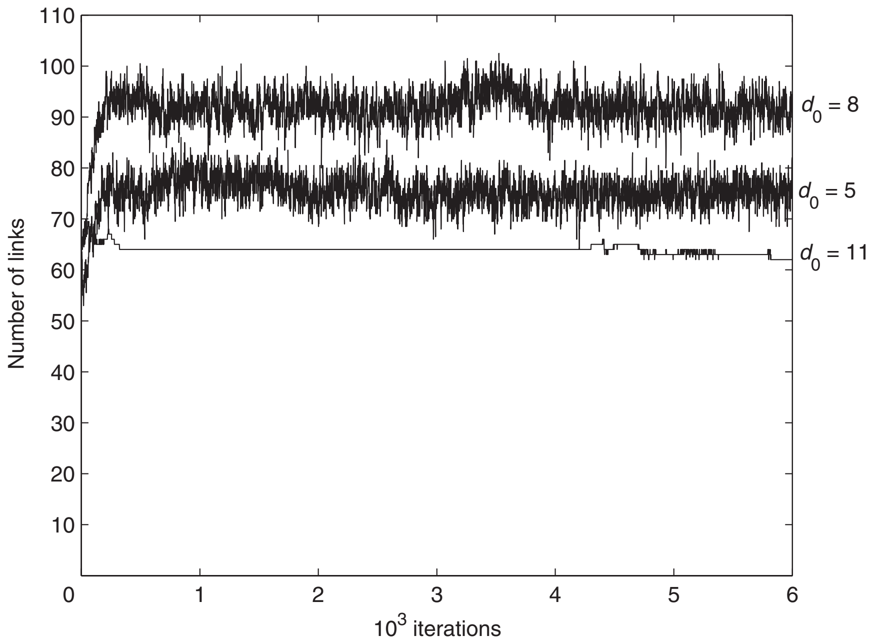

Fig. 9. Total number of links in a group of 20 robots ($d_{\max} = 15$ and $d_0 \in \{5, 8, 11\}$).

local coordinates were used to determine the occlusion cone caused by that robot with respect to the robot at the origin of that local coordinates. Any partially occluded robot was taken as a fully occluded robot. The following scenario was applied: The leader is given a trajectory and as the leader starts navigation, the rest of the group follows the leader under the local steering strategy. Local steering was achieved through the use of Eq. (19) with the gain $\gamma = 0.2$.

In the first simulation, a group consisting of 20 robots was initialized to the locations given in top of Fig. 7. The snapshots of the simulation can be seen in Fig. 7, where the trajectory of the leader is shown by solid lines. As soon as the simulation started with $d_0 = 8$, the local steering strategy forced the robots to form a more compact group. The compactness of the group was preserved until the end of the navigation, and group connectivity was maintained as expected.

The second simulation was realized with $d_0 = 11$. In this simulation there were three more robots, which initially were not connected to the group. Figure 8 shows the snapshots of the simulation. Since the desired distance was larger, the group occupied a wider area but kept its connectivity. Although the extra three robots were not connected to the group at the beginning, they join the group during navigation as soon as a robot from the group becomes as close as $d_{\max}$ to them. In other words, once an extra robot enters the sensing range, it becomes member of the group and moves accordingly during the rest of the navigation.

The last part of the simulation aims to assess the impact of d_0 on connectivity. Simulations with 20 robots, initially located as in Fig. 7, were run using three different d_0 values, namely $d_0 = 5, 8$, and 11. In all the cases, the number of links first increases rapidly and then fluctuates around an average value for the rest of the trajectory, as seen in Fig. 9. The ripples on the total number of links for $d_0 = 5$ and 8 stems from heavy occlusions. We conclude that the lower number of connections for the former value of d_0 is due to occlusions.

6. Conclusions

This work is on assuring connectivity of a navigating group of simple mobile robots. The methodology presented does not require communication between robots. Rather, a local steering strategy, which uses only the position information of the neighbors, is employed to sustain the connectivity of group members. The limited-range sensors are modeled as having angular and radial position measurement errors in order to be more realistic. Moreover, the robots may be occluded by other robots. Hence, the methodology accounts for the most fundamental difficulties in real-life implementation.

This study demonstrates that once the robots start navigation as a connected group, the steering strategy assures their connectivity without any risk of deadlock. The fact that no communication or hierarchy among robots is required allows new members to be accepted into the group easily. The success of the methodology is illustrated by simulations.

Currently the main drawbacks of the proposed scheme are the assumption of omni-directional robots and the case of failure of a robot (such as losing its motion capability). Although connectivity is always preserved, the navigation of the group might be suspended in the case robot fails. Elimination of these drawbacks could be explored in future work.

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